Exercise 31

Suppose that the cost (in dollars) for a company to produce x pairs of a new line of jeans is

$$C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3$$

(a) Find the marginal cost function.

(b) Find $C'(100)$ and explain its meaning. What does it predict?

(c) Compare $C'(100)$ with the cost of manufacturing the 101st pair of jeans.

Solution

Part (a)

Take the derivative of the cost function to get the marginal cost function.

$$\begin{aligned}\frac{dC}{dx} &= \frac{d}{dx}(2000 + 3x + 0.01x^2 + 0.0002x^3) \\ &= 0 + 3(1) + 0.01(2x) + 0.0002(3x^2) \\ &= 3 + 0.02x + 0.0006x^2\end{aligned}$$

Part (b)

Plug in $x = 100$ to get $C'(100)$.

$$C'(100) = 3 + 0.02(100) + 0.0006(100)^2 = 3 + 2 + 6 = 11 \ \frac{\$}{\text{pair of jeans}}$$

This is the rate that the cost is increasing as the 100th pair of jeans is made; it's also an estimate for the cost of the 101st pair of jeans.

Part (c)

To obtain the actual cost of the 101st pair of jeans, subtract $C(100)$ from $C(101)$.

$$\begin{aligned}C(101) - C(100) &= [2000 + 3(101) + 0.01(101)^2 + 0.0002(101)^3] \\ &\qquad - [2000 + 3(100) + 0.01(100)^2 + 0.0002(100)^3] \\ &= (1611.07) - (1600) \\ &= \$11.07\end{aligned}$$

Use the percent difference formula to see how good the estimation in part (b) is.

$$\frac{11 - 11.07}{11.07} \times 100\% \approx -0.63234\%$$

Therefore, $C'(100)$ underestimates the actual cost of the 101st pair of jeans by about 0.63%.